Take Home Quiz 9

MATH 221-02

DUE: Monday, April 27, 2015 Chapters 1-6

Use MATLAB to assist with any and all of the problems of this take-home quiz. **TURN IN ALL WORK!** This includes turning in all of your handwork, your answers, and the corresponding MATLAB code. Organize the MATLAB code with the problem number and letter so that I can understand what you have done. No organized work, no credit.

This is not a hard assignment to do. The hardest parts will be: to log on to a computer that runs matlab; and to print out your work!. Lab 1 is available on the class website at

`http://www.math.montana.edu/~parker/courses/M221/lab1.pdf`

and gives some tips on logging into a university computer in Reid or Roberts. To do the printing, you may simply want to cut and paste your work into a text file, email it to yourself, and then you can print it out at your convenience somewhere else.

Lab 1 gives easy to understand examples of the following operators and functions used by MATLAB:

=,',*,+,\,norm(),eye(),rref(), det(),inv(),rank(),null(),eig().

All the commands that you need to use to complete this take home quiz are in this list. For MATLAB help, type **help** and then the name of the function for which you need assistance. For example, type: **help inv** to see how to use the **inv()** command.

1. Consider the following matrix

$$A = \begin{pmatrix} -1 & 1 & 2 & 0 \\ 4 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 13 & 4 & 2 \end{pmatrix}.$$

   (a) Without actually trying to find the inverse of this matrix, show that it is invertible.

   (b) Find A^{-1}.

   (c) Use the inverse matrix that you found in (b) to find a solution to the system

$$\begin{pmatrix} -1 & 1 & 2 & 0 \\ 4 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 13 & 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}.$$

   (d) Now use Gauss Elimination to solve the matrix system in #1c. Do you get the same answer as you got using the inverse?

2. Consider the big, ugly, nasty matrix

$$C = \begin{pmatrix} -7 & -9 & -4 & 5 & 3 & -3 & -7 \\ -4 & 6 & 7 & -2 & -6 & -5 & 5 \\ 5 & -7 & -6 & 5 & -6 & 2 & 8 \\ -3 & 5 & 8 & -1 & -7 & -4 & 8 \\ 6 & -8 & -5 & 4 & 4 & 9 & 3 \end{pmatrix}$$

   and let $\vec{b}$ be the 5×1 vector $\vec{b}^T = [1 \;\; 1 \;\; 1 \;\; 1 \;\; 1]$.

(a) Use Gauss Elimnation to attempt to solve the matrix system $C\vec{x} = \vec{b}$ for the 5×1 vector $\vec{b}^T = [1 \ \ 1 \ \ 1 \ \ 1 \ \ 1]$. Report what MATLAB produces.

(b) Let $\vec{x}^{MATLAB}$ be the answer that MATLAB gave you in (a). Verify that $\vec{x}^{MATLAB}$ is really a solution by calculating $||C\vec{x}^{MATLAB} - b||$ (the vector $C\vec{x}^{MATLAB} - b$ is called the *residual*). What value do you get? How does this value verify that x^{MATLAB} is really a solution to $C\vec{x} = \vec{b}$?

3. Consider the big ugly nasty matrix from #2.

(a) Find a basis for the null space of C, $N(C)$.

(b) Find a basis for the column space of C, $C(C)$. One way to do this is to have MATLAB find rref(C).

(c) Use your answers to (a) and (b) and the theory that we have developed in class to find the dimensions of the Four Fundamental spaces: $C(C)$, $N(C)$, $R(C) = C(C^T)$ and $N(C^T)$. Justify your answer.

(d) In #2, MATLAB returned a single solution to $C\vec{x} = \vec{b}$ for $\vec{b}^T = [1 \ \ 1 \ \ 1 \ \ 1 \ \ 1]$. Use your results to (a)-(c) of this problem and the Fredholm Alternative to determine how many solutions there are to $C\vec{x} = \vec{b}$. If you say none, then explain what MATLAB actually gave you in #2. If you say 1, explain why your answer is correct. Or if you say there are an infinite number of solutions, then explain why this answer is correct and write the solution as a particular solution plus a homogeneous solution.

4. You want to understand the relationship between age and income. You collect data from 4 friends (Andy, Becky, Carla, and Dwayne) and get the data in the following table.

	Age (years)	Income (thousands of dollars)
Andy	19	18
Becky	81	21
Carla	31	52
Dwayne	49	61

(a) Graph these data by performing the following commands in MATLAB:

```
figure(1)
Age = [19 81 31 49]'
Income = [18 21 52 61]'
plot(Age,Income,'*')
xlabel('Age (years)')
ylabel('Income (thousands of dollars)')
title('Cash flow of my four friends')
```

Do not print out this graph yet (see below).

(b) To fit a quadratic model $Income = a + b(Age) + c(Age)^2$ to these data, set up the matrix system $A\vec{x} = \vec{b}$ so that $\vec{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$. Carefully identify A and $\vec{b}$.

(c) Use the Fredholm Alternative to determine whether you should expect 0, 1 or an infinite number of solutions to the system $A\vec{x} = \vec{b}$ from #4b. Explain.

(d) Use MATLAB to apply Gauss Elimination to attempt to solve the matrix system $A\vec{x} = \vec{b}$. Report what MATLAB produces.

(e) Let $\vec{x}^{MATLAB}$ be the answer that MATLAB gave you in #4d. Verify that $\vec{x}^{MATLAB}$ is really a solution by calculating $||A\vec{x}^{MATLAB} - b||$. What value do you get? Does this value verify or refute that x^{MATLAB} is really a solution to $A\vec{x} = \vec{b}$? Explain.

(f) How many least squares solutions do you expect to the system $A\vec{x} = \vec{b}$? Explain.

(g) Find the least squares solution to the system $A\vec{x} = \vec{b}$ by implementing the following commands in MATLAB:

```
B=A'*A
Atb = A'*Income
xls = B \ Atb
```

Be sure to provide in your report the output for each of these commands.

(h) Compare the least squares solution you found in #4g to the solution MATLAB found in #4d. Explain what you see. So what did MATLAB actually produce in #4d. Does this worry you? Explain.

(i) Add the graph of the least squares quadratic to the graph you made in #4a by implementing the following commands.

```
figure(1)
hold on
ax = axis  % Gets the size of the current graphing window
agegrid = ax(1):.1:ax(2);
quad = xls(1) + xls(2)*agegrid + xls(3)*agegrid.^2;
plot(agegrid,quad,'r')
title('Quadratic fit to the cash flow of my four friends')
```

Print out this graph and turn it in with your report.

5. Let $F = \begin{pmatrix} -7 & -9 & -4 & 5 & 3 \\ -4 & 6 & 7 & -2 & -6 \\ 5 & -7 & -6 & 5 & -6 \\ -3 & 5 & 8 & -1 & -7 \\ 6 & -8 & -5 & 4 & 4 \end{pmatrix}$ and let $B = F^T F$.

(a) Find the eigenvectors and eigenvalues of B.

(b) Form the matrix S with eigenvectors as columns (MATLAB gave you this matrix in #5a), and calculate $S^{-1}BS$. What matrix do you get? Why should this result be expected?

(c) Based on your answer to #5a, is B invertible? Explain why or why not.

(d) Find the single basis vector of $N(B)$ using MATLABs **null()** command.

(e) Determine the eigenpair from #5a whose eigenvector is the same vector as your answer to #5d.

(f) Explain why the eigenvector from #5e is the same as the basis vector of $N(B)$ from #5d.