

Exam 1

STAT422

February 18, 2011

Instructions: This exam is worth 100 points. SHOW ALL WORK! No work, no credit.

1. (22 pts) Circle True or False to indicate the validity of the following statements. You do not need to justify your answer. In all of the following, assume that X_1, \dots, X_n is a SRS from some population of interest with mean μ and variance σ^2 .
 - (a) True or False: $E(\sum_{i=1}^n X_i) = \sum_{i=1}^n E(X_i)$ only when the data X_i are independent.
 - (b) True or False: $Var(X_1 + 2X_2) = Var(X_1) + 2Var(X_2)$ only when the data X_i are independent.
 - (c) True or False: The sum of χ^2 rvs has a χ^2 distribution when the rvs are independent.
 - (d) True or False: If S^2 is the sample variance, then $\lim_{n \rightarrow \infty} Var(S^2) = 0$.
 - (e) True or False: For the normal distribution $N(\mu, \sigma^2)$, \bar{X} is the MVUE for μ .
 - (f) True or False: $\frac{1}{n} \sum_i (X_i - \bar{X})^2$ is an unbiased estimator for σ^2 .
 - (g) True or False: Using $p = .5$ maximizes the variance of a binomial random variable.
 - (h) True or False: A t statistic with n degrees of freedom is defined as $t = \frac{Z}{\sqrt{U}}$ where $Z \sim N(0, 1)$ and $U \sim \chi^2(n)$.
 - (i) True or False: The MOM estimator for $E(X^3)$ is $\frac{1}{n} (\sum_{i=1}^n X_i)^3$.
 - (j) True or False: You measured the temperature on 5 randomly chosen days in January, 2011 in Bozeman. Assuming the data are normal, a 95% CI for the true mean January temperature is $\bar{X} \pm 1.96 \frac{S}{\sqrt{n}}$.
 - (k) True or False: A 95% confidence interval for $\mu_1 - \mu_2$ is $[-6, -1]$. Thus, with 95% confidence, $\mu_2 > \mu_1$.
2. (6 pts) State the Central Limit Theorem. Be sure to state all assumptions, and give a proper conclusion.
3. (6 pts) State the Weak Law of Large Numbers. Be sure to state all assumptions, and give a proper conclusion.

4. (22 pts) On February 5, 2011, the Bozeman Daily Chronicle reported that out of 224 bison captured while attempting to exit Yellowstone National Park, 76 tested positive for brucellosis, a disease which causes animals to abort their young.
- (a) (2 pts) Use an unbiased estimator to give an estimate of the true proportion of bison who have brucellosis.
- (b) (6 pts) Give a 95% CI for the true proportion of Yellowstone bison who have brucellosis.
- (c) (4 pts) Give a conclusion of this confidence interval in terms of the problem.
- (d) (4 pts) The Chronicle reported that “about half of the Park’s bison carry the disease.” Does your 95% CI refute this claim. Why or why not?
- (e) (6 pts) What TWO things must you assume about the sample of bison in order for your conclusions in 4c and 4d to be valid? Do these two assumptions appear to be satisfied? Explain.

5. (18 pts) Suppose that X_1, X_2 are a SRS from $N(\mu, \sigma^2)$.

- (a) Give the distribution of $\frac{(Y_1 - Y_2)^2}{2\sigma^2}$, and explain why your answer is correct. (*Hint:* What is the distribution of $(Y_1 - Y_2)$?)

- (b) Show that $\frac{(Y_1 - Y_2)^2}{2\sigma^2}$ is a pivotal quantity for σ^2 .

- (c) Apply the pivotal method to $\frac{(Y_1 - Y_2)^2}{2\sigma^2}$ to construct a CI for σ^2 .

6. (12 pts) Let X_1, \dots, X_n be a SRS from an exponential distribution with parameter β .

(a) (8 pts) Prove that $\sum_i X_i$ is sufficient for β . Justify each step in your proof.

(b) (4 pts) Find the MVUE for β . Specify which theorem justifies your answer.

7. (14 pts) Suppose that X_1, \dots, X_n are a SRS from a Poisson distribution with parameter λ .

(a) (10 pts) Find the MLE of λ . Be sure to show that the statistic you find is a maximum.

(b) (4 pts) Find the MLE for $\lambda^2 + \lambda$. Justify your answer.