Homework #1

Due: Friday, January 27, 2017

Sir Francis Galton (Natural Inheritance, 1889) described the Central Limit Theorem as (from www.wikipedia.org):

> I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the "Law of Frequency of Error". The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement, amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshaled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to have been latent all along.

1. Do exercise 7.10, parts (b)-(d). Show your work. Use either tables in the back of the book or R to do the probability calculations.
2. Do exercise 7.12.
3. Do exercise 7.20. These are proofs, justify each of your steps, and make sure to check any assumptions.
4. Do exercise 7.42. As you answer this question, explicitly give the distribution of $\bar{X}$.
5. Do exercise 7.58.
6. Suppose that $Y \sim \text{Bin}(n = 5,\, p = 0.1)$.
   (a) Use R's pbinom( ) function to calculate $P(Y < 1)$.
   (b) Which normal distribution approximates this binomial distribution?
   (c) Use R's pnorm( ) function to use the normal approximation to the binomial to calculate $P(Y < 1)$.
   (d) The normal approximation is not particularly good. Why?
7. Suppose that $Y \sim \text{Bin}(n,\, p = 0.1)$. Find the smallest sample size n so that the exact binomial probability $P(Y < 1)$ and the normal approximation differ by less than 0.01. Show your work.
8. Do exercise 7.80.
9. Do exercise 7.84 (Exercise 7.15 is similar to this one).
10. (a) State the Central Limit Theorem.
    (b) Prove the Central Limit Theorem.