Homework #5

Due: Friday, March 10, 2017

William Thomson (aka, Lord Kelvin) (1824 - 1907):

> When you can measure what you are speaking about and express it in numbers you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of an unsatisfactory kind.

1. Do exercise 9.70. Yes, this really is that easy.
2. Do exercise 9.74. Start this problem by finding EY. Because EY is a function of θ, solve for θ to get a MOM estimator for θ.
3. Do exercise 9.80bcd (we already did part a in class). *Hint for part d*: Start with your answer to part a then use the invariance property of MLEs.
4. Consider a random sample of size n from $EXP(\theta)$. Find the MLE of θ. Be sure to do either the 1st or 2nd derivative test.
5. Do exercise 9.81. *Hint:* Use the invariance property of MLEs.
6. Consider a random sample of size n from $N(\mu, \sigma^2)$ with σ^2 known. Find the MLE of μ. Be sure to do either the 1st or 2nd derivative test. *Hint:* See Example 9.15 that deals with the case when both μ and σ are unknown.
7. Do Exercise 9.96. *Hint:* Use the invariance propoerty of MLEs.
   (a) Derive the MLE for σ.
   (b) Derive the MLE for $Var(S^2)$ where S^2 is the usual unbiased estimator of σ^2.
8. Do Exercise 9.97a (we already did part b in class).
9. Consider a random sample $y_1, ..., y_n \sim$ Geometric(p) distribution as in Exercise 9.97. Find the MLE of $Var(Y)$.
10. **REQUIRED for math/stat graduate students. Extra Credit for everyone else.** Do Exercise 9.94, which proves a special case of the Invariance Property of MLEs. Specifically, when $t(\cdot)$ is a 1-1 function and $\hat{\theta}$ is the MLE of θ, you will prove that $t(\hat{\theta})$ is the MLE of $t(\theta)$. Although short, this is only an easy proof after you see how it is done. There are lots of places to find this proof, so find it, then write it in a way that makes sense to you.

Additional problems: Make sure you can do 9.80a and 9.97b (that we did in class), including the 1st or 2nd derivative tests.