Exam 2

MATH 221-2

March 31, 2014 Chapters 1-4

1. (28 pts, 2 pts each) Circle **T** or **F** indicating whether each of the following statements are True or False. You do not need to justify your answers.

(a) **T** **F** For any matrix A, $\mathcal{C}_A$ and $\mathcal{N}(A^T)$ are orthogonal vector spaces.

(b) **T** **F** If $\dim(\mathcal{N}(A)) = 1$, then $A\boldsymbol{x} = \mathbf{0}$ has an infinite number of solutions.

(c) **T** **F** If $A = \begin{bmatrix} | & | & & | \\ \boldsymbol{v}_1 & \boldsymbol{v}_2 & \ldots & \boldsymbol{v}_n \\ | & | & & | \end{bmatrix}$ then $\text{span}(\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_n) = \mathcal{C}_A$.

(d) **T** **F** If A is a 7×8 matrix then A^{-1} is a 8×7 matrix.

(e) **T** **F** If two non-zero vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal, then $\boldsymbol{u}$ and $\boldsymbol{v}$ are linearly independent.

(f) **T** **F** For a 2×3 matrix A, if the general solution to $A\boldsymbol{x} = \boldsymbol{b}$ is $\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + z \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ (z is a scalar), then there are non-zero solutions to $A^T\boldsymbol{y} = \mathbf{0}$.

(g) **T** **F** If $A = \begin{bmatrix} -1 & 1 & 3 \\ 0 & 0 & 0 \\ 2 & 1 & -6 \end{bmatrix}$ then $\dim(\mathcal{C}_A) = 2$.

(h) **T** **F** If $A = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$ then $A\boldsymbol{x} = \boldsymbol{b}$ has an infinite number of solutions for any $\boldsymbol{b} \in \mathcal{C}_A$.

(i) **T** **F** Any set of n linearly independent vectors in $\Re^n$ is a basis for $\Re^n$.

(j) **T** **F** If $\mathcal{V} = \text{span}(\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_n)$ then $\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_n$ are a basis for $\mathcal{V}$.

(k) **T** **F** If the columns of A are linearly independent, then there is always at least 1 solution to $A\boldsymbol{x} = \boldsymbol{b}$.

(l) **T** **F** If A is a 3×2 matrix and $A\boldsymbol{x} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ has only one solution then $\mathcal{N}(A) = \{\mathbf{0}\}$.

(m) **T** **F** If A is a 10×5 matrix, then $\text{rank}(A) = 10 - \dim(\mathcal{N}(A^T))$.

(n) **T** **F** If $\mathcal{N}(A) = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$, then $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \in \mathcal{R}_A$.

2. (4 pts) Perform the following matrix-vector multiplication: $A\boldsymbol{x} = \begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = ?$

3. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \end{pmatrix}$.

(a) (16 pts) Solve the system of linear equations

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 4 \\ 2x & + & 4y & & & = & 2 \end{array}.$$

SHOW YOUR WORK.

(b) (5 pts) Based on the work you did in #3a, give a basis for $\mathcal{C}_A$. Justify your answer.

(c) (5 pts) Based on your answer to #3a, give a basis for $\mathcal{N}(A)$. Justify your answer.

(d) (5 pts) Based on your answer to #3a, is $\begin{bmatrix} 4 \\ 2 \end{bmatrix} \in \mathcal{C}_A$? Explain why your answer is correct.

(e) (5 pts) For any non-zero $\boldsymbol{b} \in \Re^2$ that one might choose, explain why finding only 1 solution to $A\boldsymbol{x} = \boldsymbol{b}$ is impossible.

(f) (5 pts) For any $\boldsymbol{b} \in \Re^2$ that one might choose, explain why finding no solutions to $A\boldsymbol{x} = \boldsymbol{b}$ is impossible.

(g) (5 pts) Based on your answers to #3e-3f, for any $\boldsymbol{b} \in \Re^2$, how many solutions will there always be for $A\boldsymbol{x} = \boldsymbol{b}$? Explain.

4. (12 pts) This is a 3-part question. Answer all 3 parts to get full credit: (i) What is a homogeneous solution of a matrix A? (ii) How do homogeneous solutions of A relate to the $\mathcal{N}(A)$? (iii) Why are homogeneous solutions important when solving $A\boldsymbol{x} = \boldsymbol{b}$?

5. (10 pts) For any $m \times n$ matrix A, prove that $\mathcal{N}(A)$ is a vector space. Explicitly state the conditions that you are checking, and justify each step in your proof.