Tracking Eye Motion from Retinal Scan Data with a Map Seeking Circuit

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Joint work with

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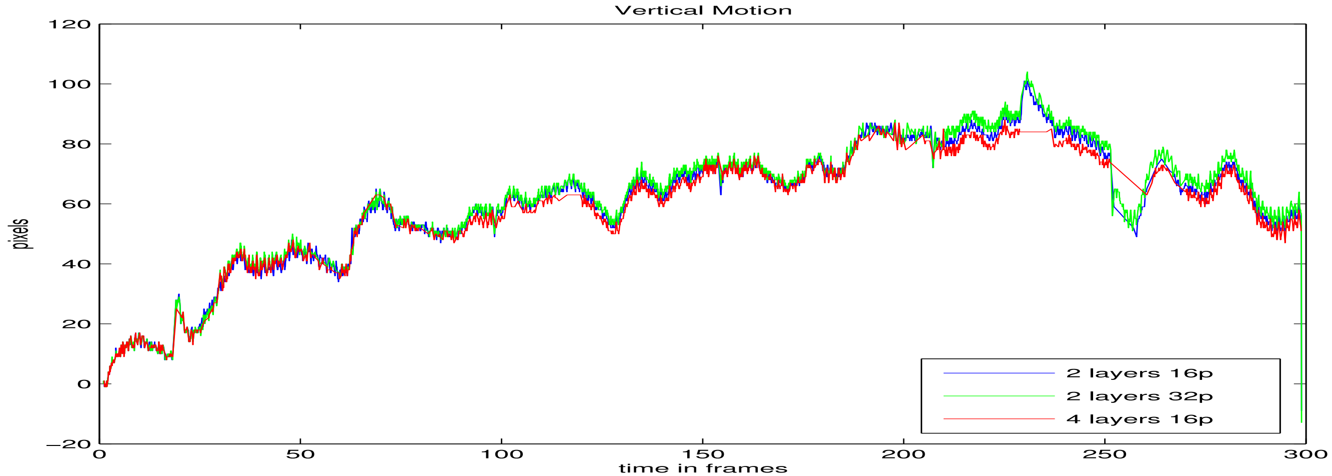
Outline

- Show and Tell (Results)
  - Estimating Eye Motion from AOSLO data
  - Image Registration

- The Algorithm which finds the motion: Map-Seeking Circuit (MSC)

Results: Estimated Motion from AOSLO Data



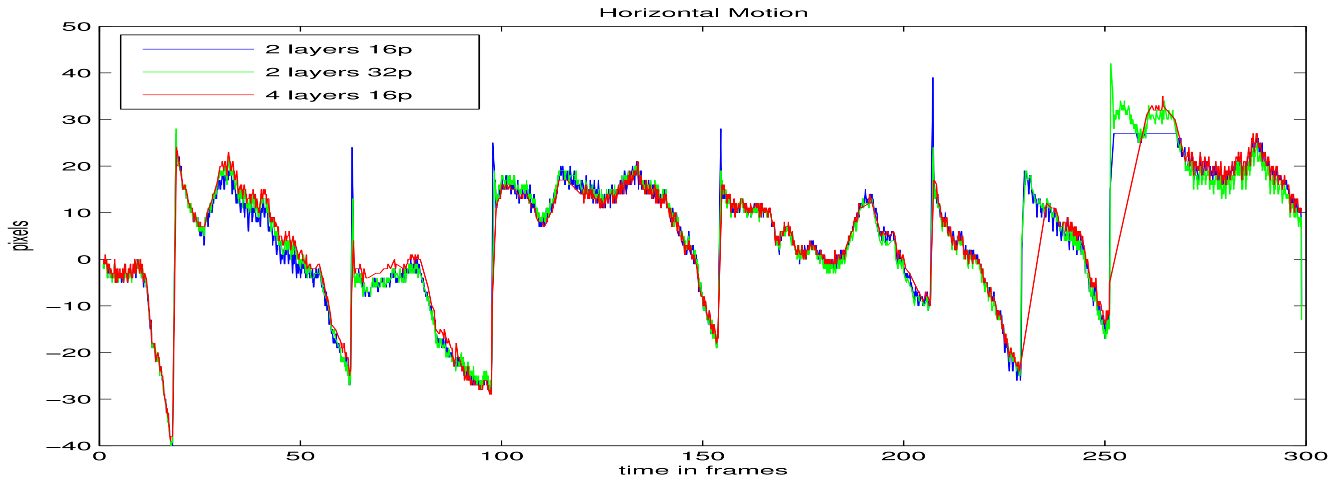
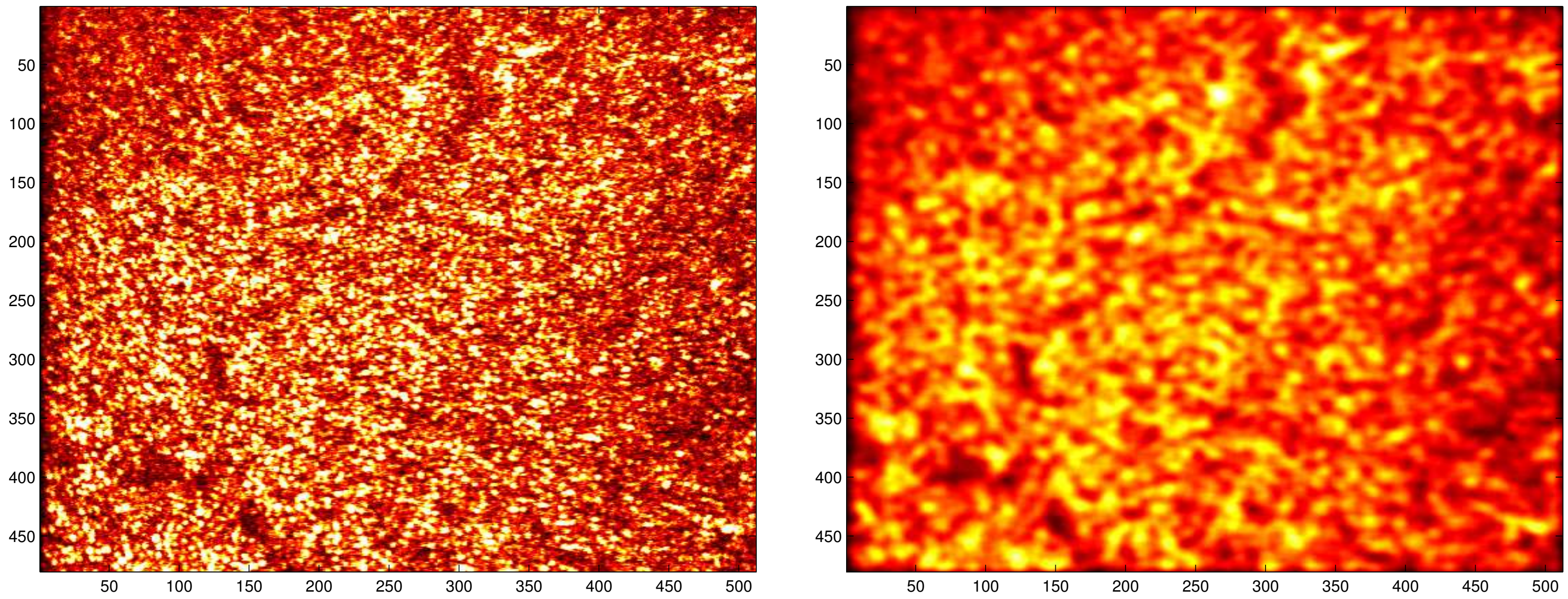


Image Preprocessing

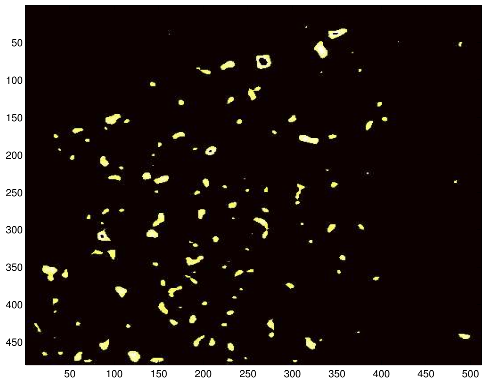
1. Smooth the data with a Gaussian kernel to reduce the effect of noise and and amplitude variation.

Raw Smoothed

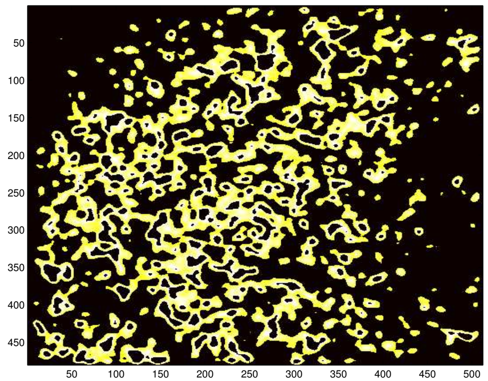


2. Break the smoothed image into "channels".

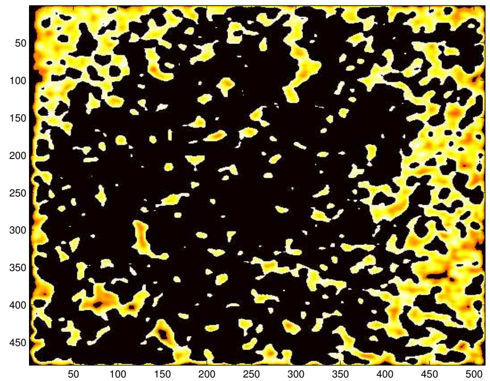
High



Medium

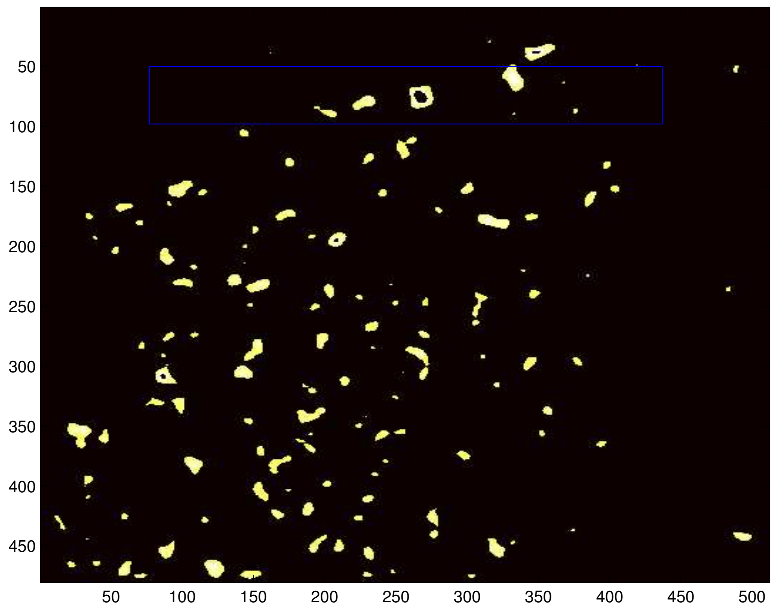


Low



3. Determine the eye motion across “patches”.

This yields HIGH RESOLUTION of the motion: We can calculate 16 - 32 estimates of the motion per frame, which is about 480 - 960 estimates per second.



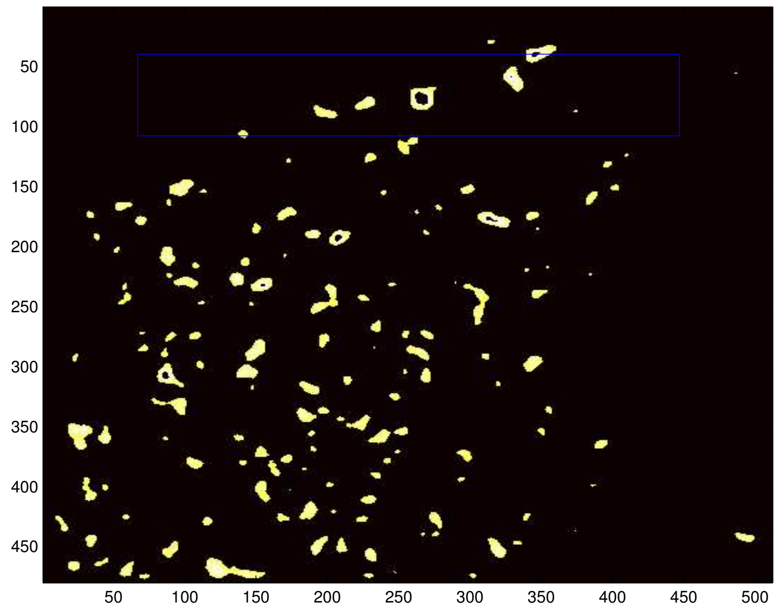
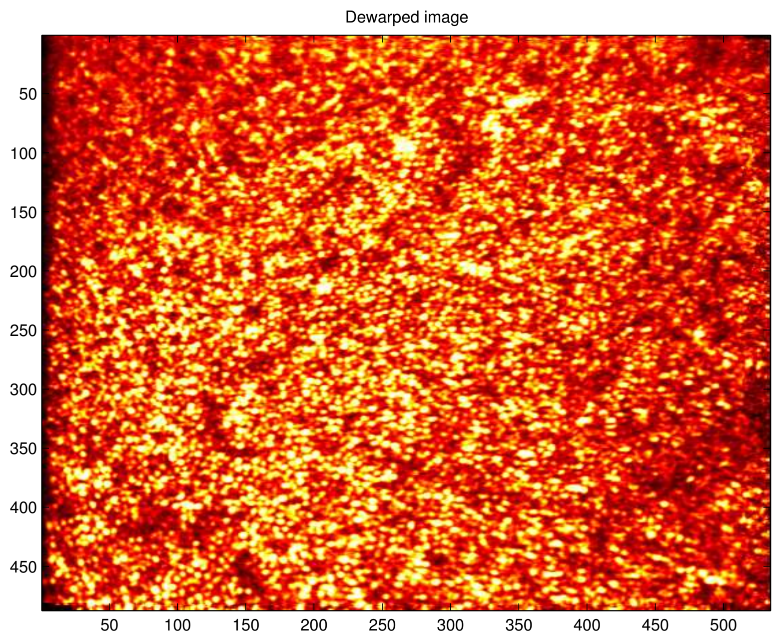


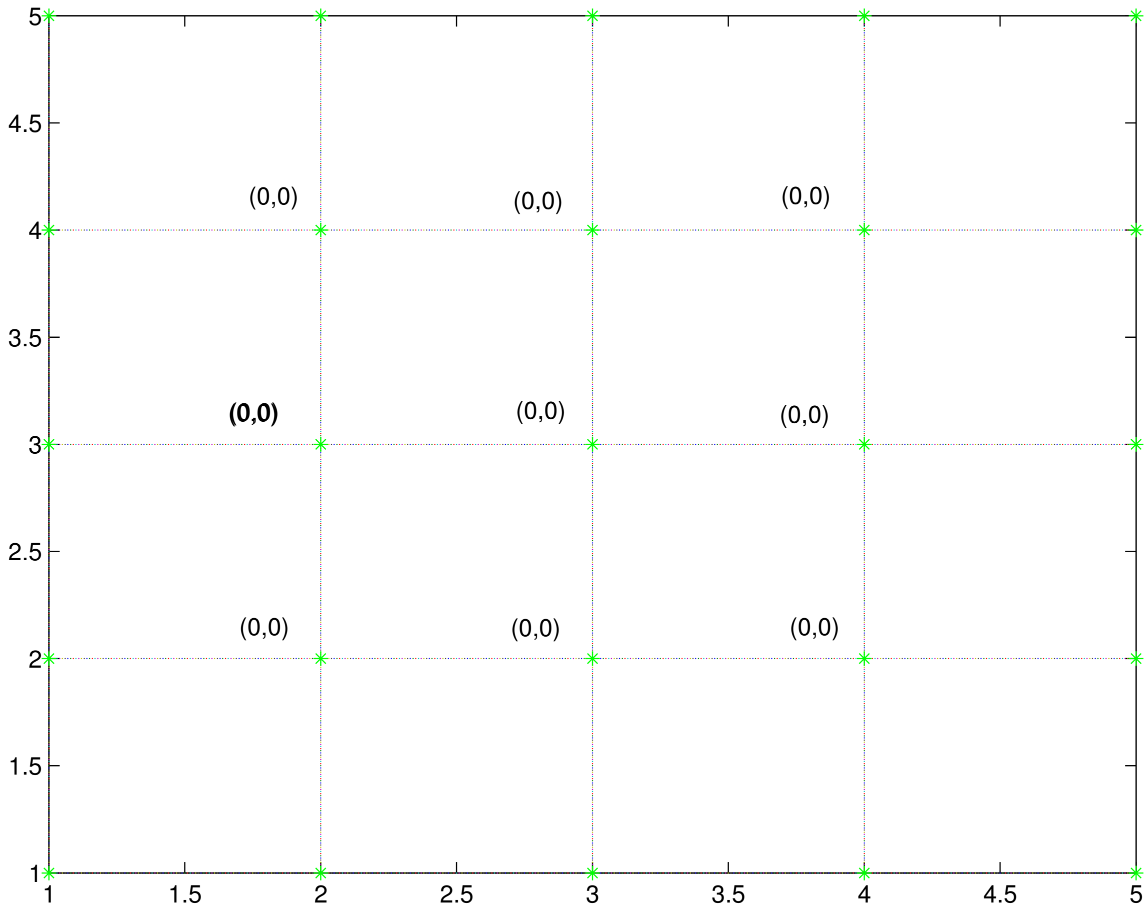
Image Registration

- Dewarp each frame of the AOSLO video

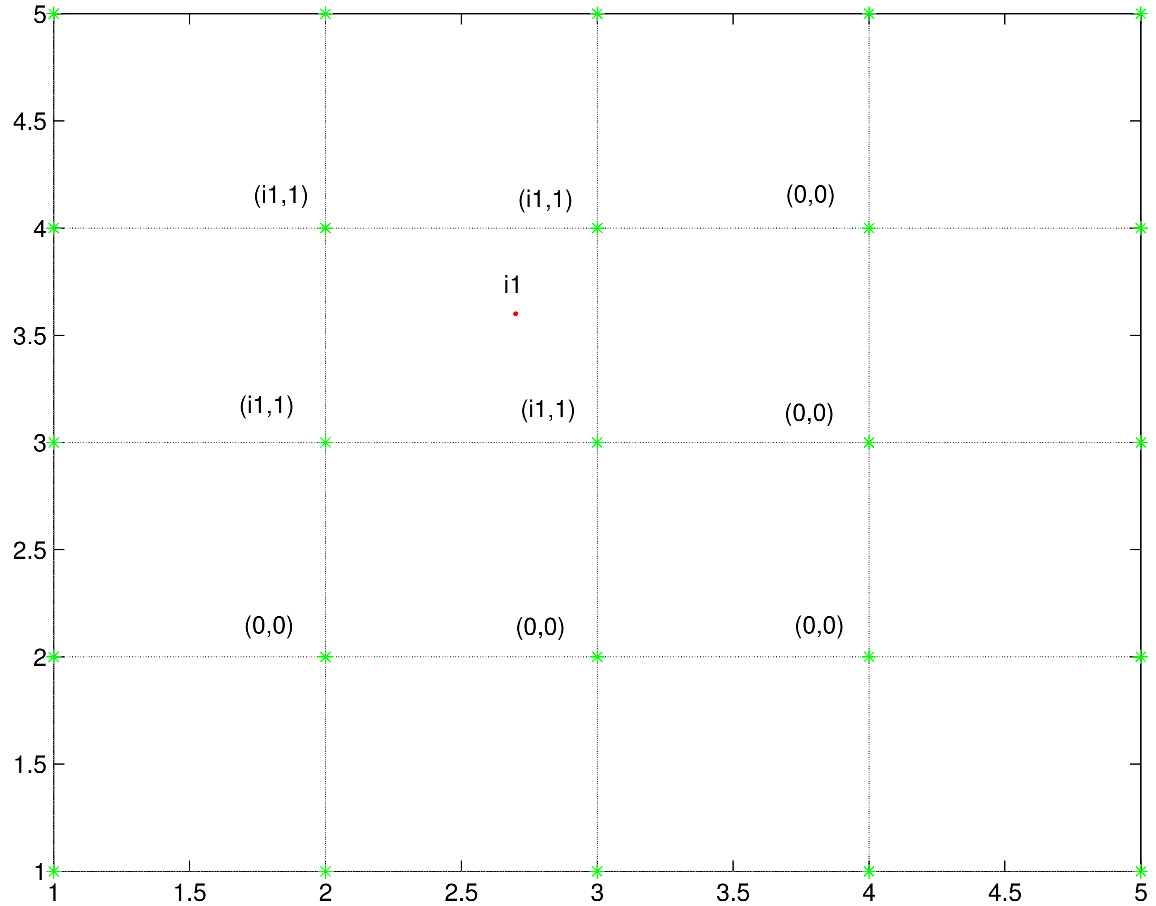
- Add each dewarped frame to create a mosaic or montage



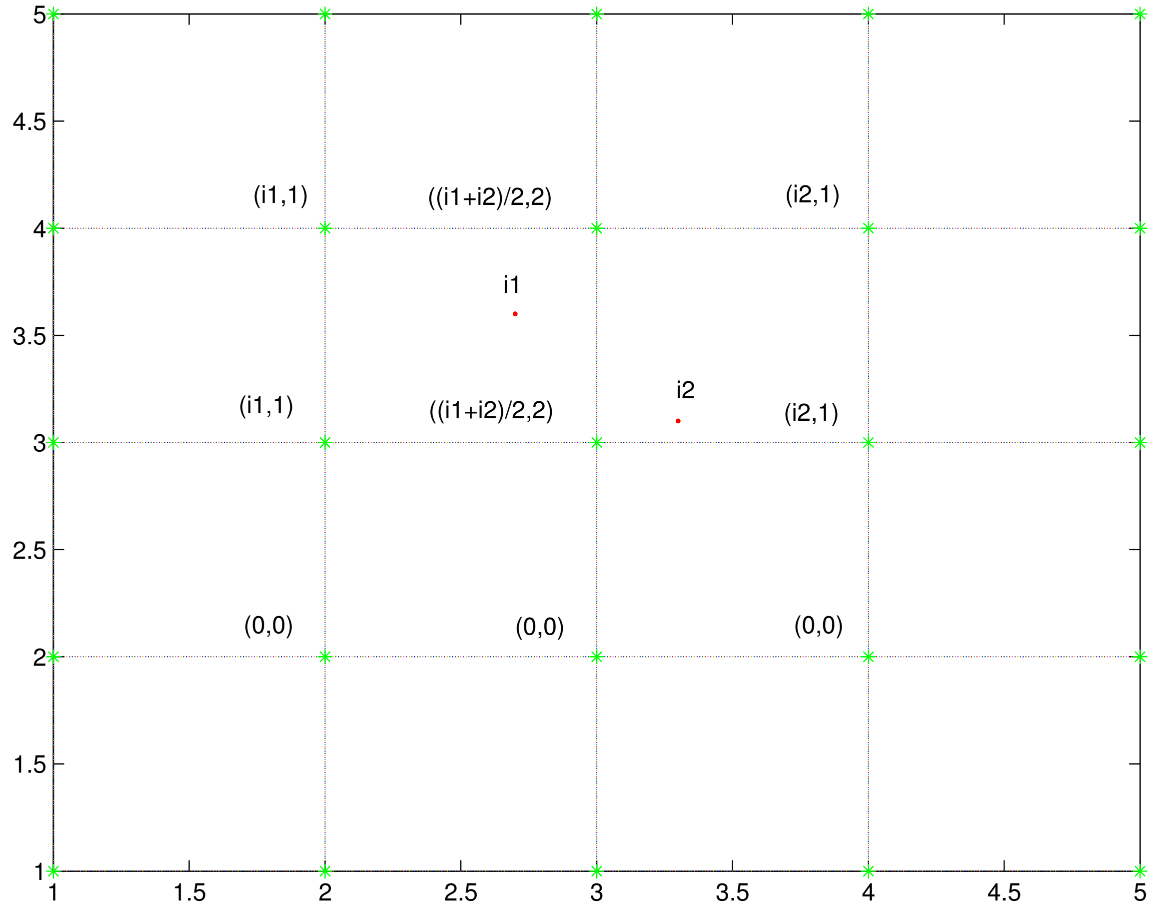
How to add frames: Initialize montage with (intensity=0,weight=0)



How to add frames: Map the first pixel, update the montage



How to add frames: Map the next pixel, update the montage ...



MAP-SEEKING CIRCUIT ALGORITHM (MSC)

Model images E, E' as vectors in Hilbert spaces $\mathcal{H}$, $\mathcal{H}'$, respectively. Given transformation $T : \mathcal{H} \to \mathcal{H}'$, define the correspondence between E and E' associated with the transformation T to be the inner product

$$\langle T(E), E' \rangle_{\mathcal{H}'}.$$

Goal: Find T which maximizes correspondence from linear transformations of form

$$T = T^{(L)}_{i_L} \circ \cdots \circ T^{(2)}_{i_2} \circ T^{(1)}_{i_1},$$

where for each "layer" ℓ between 1 and L, we have $i_\ell \in \{1, 2, \ldots, n_\ell\}$.

For example, we can let T_{i_1} be some horizontal translation of the image E and let T_{i_2} be some vertical translation of the image E.

ADVANTAGE of MSC over Cross-Correlation: MSC can include other transformations such as rotations, dilations, shear, compression, ...

SYNOPSIS: A Map-Seeking Circuit finds a solution to the discrete optimization problem

$$(i_1^*, \ldots, i_L^*) = \text{arg} \max_{1 \le i_\ell \le n_\ell} \left\langle T_{i_L}^{(L)} \circ \cdots \circ T_{i_2}^{(2)} \circ T_{i_1}^{(1)}(E), E' \right\rangle_{\mathcal{H}'}$$

MSC KEY IDEA (which makes it fast)

Imbed the discrete problem in continuous constrained optimization problem. Maximize multilinear form

$$M(\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(L)}) \;=\; \langle T_{\mathbf{x}^{(L)}} \circ \cdots \circ T_{\mathbf{x}^{(1)}}(E), E' \rangle_{\mathcal{H}'},$$

where

$$T_{\mathbf{x}^{(\ell)}} = \sum_{i=1}^{n_\ell} x_i^{(\ell)} T_i^{(\ell)}$$

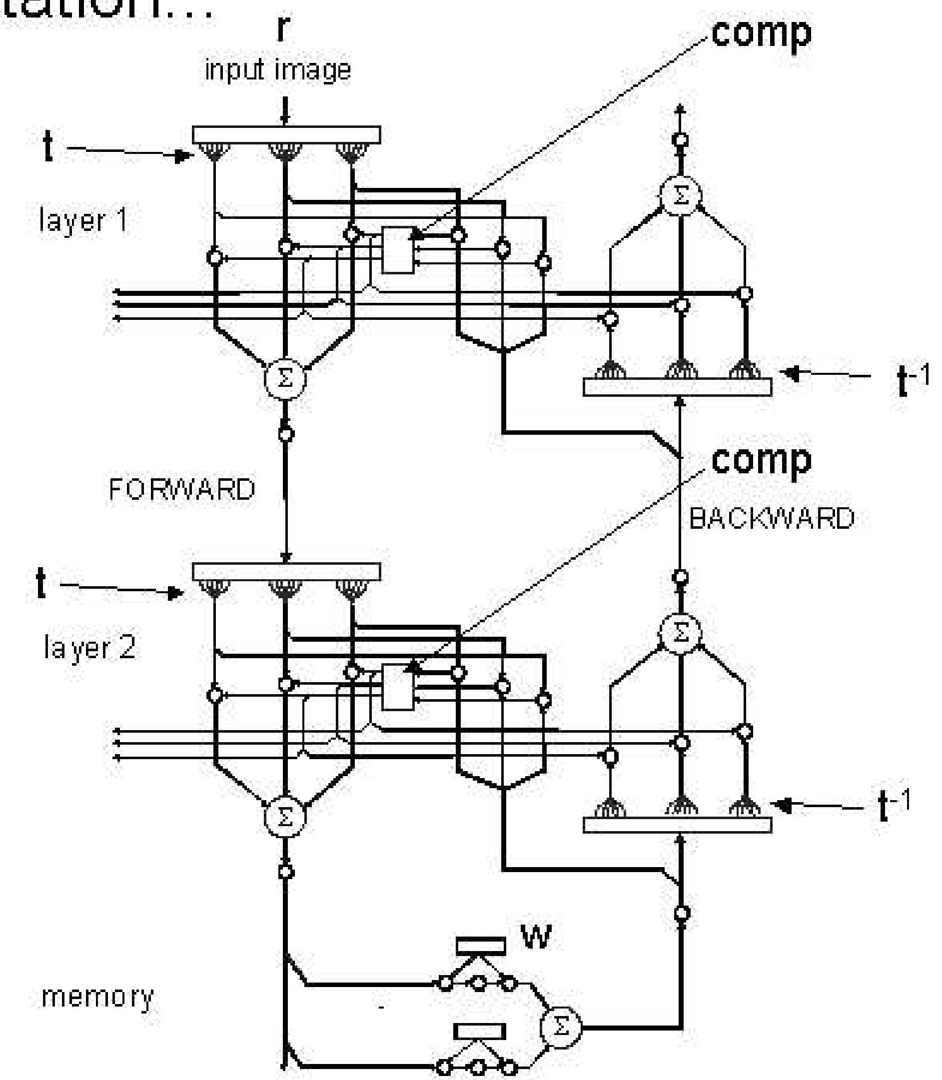
SIMPLIFYING PROPERTY: Components of gradient of M can be computed quickly and relatively cheaply via the inner product

$$\frac{\partial M}{\partial x_i^{(\ell)}} = \langle T_{i_\ell}^{(\ell)} \circ T_{\mathbf{x}^{(\ell-1)}} \circ \cdots \circ T_{\mathbf{x}^{(1)}}(E), T'_{\mathbf{x}^{(\ell+1)}} \circ \cdots \circ T'_{\mathbf{x}^{(L)}}(E') \rangle_{\mathcal{H}'}$$

where T' is the *adjoint* or *conjugate transpose* of T.

MSC can viewed as an iterative algorithm which uses this gradient info to maximize the correspondence.

Implementation...



COMPUTATIONAL COST

- Computation complexity of each MSC-type iteration is on order of the sum

$$n_1 + n_2 + \ldots + n_L,$$

  where n_ℓ is the number of transformations in layer ℓ, and L is the number of layers.

- The complexity of an exhaustive search is the product

$$n_1 n_2 \cdots n_L.$$

Convergence tends to be fast and solutions tend to be robust if the data has a "sparse encoding". Image data pre-processing can provide such a sparse encoding.

CONCLUSIONS

Using MSC,

- We can accurately track eye motion from AOSLO videos, even through saccades.

- We can identify very general transformations between AOSLO video frames, including shear and compression.

- Register AOSLO video frames to create a de-noised retinal image