

QUIZ I:MATH 274

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NAME: Solns

1. [3pts] Fill in the table below:

ODE	Order	Linear (Y/N)	Independent Variable Name	Dependent Variable Name
$\frac{d^3 y}{dx^3} + x^4 y = 1$	3	Y	x	y
$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right) + y = 1$	1	N	x	y
$t^3 \frac{d^2 x}{dt^2} + e^t x^4 = 0$	2	N	t	x

2. [5pts] Find all the values of m such that $y(x) = x^m$ is a solution of

$$2x^2 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$$

$$y' = m x^{m-1}$$

$$y'' = m(m-1) x^{m-2}$$

Eqn becomes

$$[2m(m-1) - 3m + 2] x^m = 0$$

$$2m^2 - 5m + 2 = 0$$

$$(2m-1)(m-2) = 0$$

$$m = \frac{1}{2}, 2$$

3. [4pts] The function $y(x)$ is defined implicitly by the equation below and also satisfies $y'(x) = f(x, y)$. Find $f(x, y)$.

$$xy^2 + x^3 - e^y = 0$$

Differentiate in x

$$y^2 + 2xy \frac{dy}{dx} + 3x^2 - e^y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(3x^2 + y^2)}{2xy - e^y}$$

$$f(x, y) =$$

4. [8pts] Find the explicit solution of the following (separable) initial value problem:

$$\frac{dy}{dx} = \frac{xy^2}{x^2 + 1}, \quad y(0) = 1$$

$$\int \frac{dy}{y^3} = \int \frac{x}{x^2 + 1} dx$$

$$-\frac{1}{2y^2} = \frac{1}{2} \ln(x^2 + 1) + C$$

$$\text{Use } y = 1 \text{ when } x = 0 \Rightarrow C = -1$$

$$y = \frac{1}{1 - \frac{1}{2} \ln(x^2 + 1)}$$

$$y(x) =$$