

1. [3pts] Fill in the following chart ($Y = \text{Yes}$, $N = \text{No}$)

Differential Equation	Separable	Linear	Homogeneous	Bernoulli
$\frac{dy}{dx} = \frac{x}{x+y}$	N	N	Y	N
$\frac{dy}{dx} = y + xy^3$	N	N	N	Y
$\frac{dy}{dx} = xe^y$	Y	N	N	N

2. [5pts] Find the unique solution of the following linear initial value problem:

$$y' - \frac{2}{x}y = x, \quad y(1) = 0$$

Int. Factor: $\mu(x) = \exp\left(-\int \frac{2}{x} dx\right) = \exp(-2\ln x) = \frac{1}{x^2}$

$$y_p(x) = \frac{1}{\mu} \int \mu(t) g(t) dt = x^2 \int \frac{1}{x} dx = x^2 \ln x$$

General Solution

$$y = c x^2 + x^2 \ln x$$

Initial Conditions

$$y(1) = c = 0$$

Unique solution

$$y(x) = x^2 \ln x$$

3. [6pts] Find the explicit general solution of the following homogeneous problem:

$$v = y/x$$

$$\frac{dy}{dx} = \frac{y^2 + 4xy + 2x^2}{x^2} = v^2 + 4v + 2 = G(v)$$

Then

$$\int \frac{dv}{G(v)-v} = \int \frac{dv}{v^2+3v+1} \xrightarrow{\text{partial fraction}} \int \left(\frac{1}{v+1} - \frac{1}{v+2} \right) dv = \int \frac{dx}{x}$$

Integrating

$$\ln(v+1) - \ln(v+2) = \ln x + k \quad 5$$

$$\ln\left(\frac{v+1}{v+2}\right) = \ln x + k$$

$$\left(\frac{v+1}{v+2}\right) = cx \quad 6$$

Solve for $v = y/x$ and get

$$y(x) = \frac{x(2cx-1)}{1-cx}$$

4. [6pts] Find the unique solution $y(x)$ of the following initial value problem:

$$\frac{dy}{dx} + \frac{1}{x}y = xy^{-1}, \quad y(1) = 1$$

$$v = y^{1-n} = y^2$$

$$\boxed{v' + \frac{2}{x}v = 2x}$$

linear with int. factor
 $\mu = \exp\left(\int \frac{2}{x} dx\right) = x^2$

particular soln $v_p = \frac{1}{x^2} \int x^2(2x) dx = \frac{1}{2}x^2$

$$v(x) = y^2 = \frac{c}{x^2} + \frac{1}{2}x^2 \quad \text{gen. soln.}$$

Init. Conditions

$$1 = c + \frac{1}{2} \Rightarrow c = \frac{1}{2}$$

Conclude

$$y(x) = +\sqrt{\frac{1}{2}\left(\frac{1}{x^2} + x^2\right)}$$