

QUIZ III: MATH 274

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1. [6pts] Find the general solution of the homogeneous problem:

$$y'' - 4y' + 5y = 0$$

char. poly. $P = r^2 - 4r + 5 = 0$ if $r = 2 \pm i$

$$y(x) = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x$$

2. [6pts] Find the unique solution of the following initial value problem

$$y'' - 4y' + 4y = 0, \quad y(0) = 0, \quad y'(0) = 3$$

$$P = r^2 - 4r + 4 = (r - 2)^2$$

hence general soln is

$$y(x) = c_1 e^{2x} + c_2 x e^{2x}$$

$$y'(x) = 2c_1 e^{2x} + c_2 (2x + 1) e^{2x}$$

Apply initial conditions

$$y(0) = c_1 = 0$$

$$y'(0) = c_2 = 3$$

Conclude

$$y(x) = 3x e^{2x}$$

3. [3pts] Find the general solution of the following third order differential equation

$$y''' - 8y'' + 19y' - 12y = 0$$

given that $r = 4$ is a root of its characteristic polynomial $P(r)$.

$$\begin{array}{r} r^2 - 4r + 3 = (r-3)(r-1) \\ \hline r-4 \quad \overline{r^3 - 8r^2 + 19r - 12} \\ \quad \underline{r^3 - 4r^2} \\ \quad \quad -4r^2 + 19r \\ \quad \quad \underline{-4r^2 + 16r} \\ \quad \quad \quad 3r - 12 \\ \quad \quad \quad \underline{3r - 12} \\ \quad \quad \quad \quad 0 \end{array}$$

$$P = (r-1)(r-3)(r-4)$$

$$y(x) = c_1 e^x + c_2 e^{3x} + c_3 e^{4x}$$

4. [5pts] Use the Method of Undetermined Coefficients to find a particular solution $y_p(x)$ of:

$$y'' - y' + 2y = 4x^2 + 2$$

Let $y_p = Ax^2 + Bx + C$ in the eqn and match powers of x

$$(2A) - (2Ax + B) + 2(Ax^2 + Bx + C) = 4x^2 + 2$$

$$\underbrace{2Ax^2}_4 + \underbrace{(2B - 2A)x}_0 + \underbrace{(2A - B + 2C)}_2 = 4x^2 + 2$$

Hence $A = 2, B = 2, C = 0$

$$y_p(x) = 2x^2 + 2x$$