

1. [6pts] Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ where

a) $f(t) = e^t \sin(2t)$

$$F(s) = \frac{2}{(s-1)^2 + 2^2}$$

b) $f(t) = t \cos(2t)$

$$F(s) = -\frac{d}{ds} \left(\frac{s}{s^2 + 4} \right)$$

$$F(s) = \frac{2s^2}{(s^2 + 4)^2} - \frac{1}{(s^2 + 4)} = -\frac{4 - s^2}{(s^2 + 4)^2}$$

2. [6pts] Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ where

$$F(s) = \frac{s-1}{s^2 - 4s + 8}$$

$$F(s) = \frac{(s-2)}{(s-2)^2 + 2^2} + \frac{1}{(s-2)^2 + 2^2}$$

$$f(t) = e^{2t} \cos(2t) + \frac{1}{2} e^{2t} \sin 2t$$

3. [8pts] Let $Y(s) = \mathcal{L}\{y(t)\}$ be the Laplace Transform of the solution of:

$$y'' - 3y' + 2y = e^t, \quad y(0) = 0, \quad y'(0) = 0$$

a) Determine a formula for $Y(s)$ and then write it out as a partial fraction expansion

$$(s^2 - 3s + 2)Y = \frac{1}{(s-1)}$$

$$5\text{pt} \quad Y = \frac{1}{(s-1)^2(s-2)}$$

$$Y = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{(s-2)}$$

$$Y = \frac{A(s-1)(s-2) + B(s-2) + C(s-1)}{(s-1)^2(s-2)}$$

$$s=2 \Rightarrow C=1.$$

$$s=1 \Rightarrow B=-1$$

$$s=0 \Rightarrow A=-1$$

$$6\text{pt} \quad Y = \frac{1}{(s-2)} - \frac{1}{(s-1)} - \frac{1}{(s-1)^2}$$

b) Use your result in a) to find $y(t)$.

$$8\text{pt} \quad y = e^{2t} - e^t - te^t$$