QUIZ VI:MATH 274

November 22, 2013. NAME: Solns

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1. [6pts] Use the convolution theorem to find the solution of the following initial value problem. Express your answer as a convolution integral.

$$y'' - 2y' + 2y = g(t) \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0$$

$$(s^2 - 2s + 2)Y = G(s)$$

$$Y = \frac{1}{(s-1)^2 + 1} \cdot G(s) = F(s)G(s)$$

Invert $F(s)$: $f(t) = e^t \sin t$. Convolution Thm $\Rightarrow$

$$y(t) = \int_0^t e^{\lambda} \sin\lambda \, g(t-\lambda)\, d\lambda$$

2. [6pts] Find the solution of the following IVP given the listed independent solutions $\vec{x}_k$

$$\frac{d\vec{x}}{dt} = A\vec{x} \quad , \quad \vec{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad , \quad \vec{x}_1(t) = \begin{pmatrix} e^t \\ -e^t \end{pmatrix} \quad , \quad \vec{x}_2(t) = \begin{pmatrix} e^{2t} \\ 4e^{2t} \end{pmatrix}$$

$$\vec{x}(t) = X(t)\vec{c} \qquad X = \begin{bmatrix} e^t & e^{2t} \\ -e^t & 4e^{2t} \end{bmatrix} \qquad X(0) = \begin{bmatrix} 1 & 1 \\ -1 & 4 \end{bmatrix}$$

$$\vec{c} = X(0)^{-1}\vec{x}(0) = \frac{1}{5}\begin{bmatrix} 4 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\vec{x} = \vec{x}_1 + \vec{x}_2 = \begin{pmatrix} e^t + e^{2t} \\ -e^t + 4e^{2t} \end{pmatrix}$$

3. [8pts] For the following problem, find two independent solutions $\vec{x}_1(t)$ and $\vec{x}_2(t)$ and then write out the fundamental matrix $X(t)$.

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix} \vec{x}$$

$$p(\lambda) = \begin{vmatrix} 5-\lambda & -6 \\ 3 & -4-\lambda \end{vmatrix} = \lambda^2 - \lambda - 2 = (\lambda+1)(\lambda-2)$$

Eigenvalues are $\lambda_1 = -1$, $\lambda_2 = 2$

$$A - \lambda_1 I = \begin{bmatrix} 6 & -6 \\ * & * \end{bmatrix} \qquad \vec{\xi}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$A - \lambda_2 I = \begin{bmatrix} 3 & -6 \\ * & * \end{bmatrix} \qquad \vec{\xi}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\vec{x} = c_1 \vec{x}_1 + c_2 \vec{x}_2$$

$$\vec{x} = c_1 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\underline{X} = \begin{bmatrix} e^{-t} & 2e^{2t} \\ e^{-t} & e^{2t} \end{bmatrix}$$