Math 333 (Linear Algebra) Final Exam
Friday, December 17, 2:00-3:50pm (Wil 1-117)
Handwritten notes permitted
Summary of Topics

1. Matrices $A \in \mathbb{R}^{m\times n}$
   a) Bases and dimensions for $row(A), col(A), N(A), N(A^T)$ in terms of $rank(A), nullity(A)$.
   b) General solution of $Ax = b$ especially when $N(A) \neq \{0\}$.
   c) Fredholm Alternative
   $$Ax = b \; has \; a \; solution \quad \Leftrightarrow \quad < v, b >= v^T b = 0 \; , \; \forall v \in N(A^T)$$
2. Bases for $W = span(S) = span\{u_1, u_2, \ldots u_m\} \subset V$
   a) Coordinate of v relative to a basis
   b) If $E = \{v_1, \ldots v_n\}$ is a standard basis for V then
   $$w \in W \quad \Leftrightarrow \quad (w)_E \in col(A)$$
   where A has $(u_k)_E$ are the columns of A. (See HW5, questions 4,5).
3. Inner Product Spaces
   a) Axioms defining $< u, v >$
   b) Induced norm $\| u \| = \sqrt{< u, u >}$
   c) Definitions of orthogonal and orthonormal bases.
   d) Definition of orthogonal complement $W^\perp$ and orthogonal decomposition Theorem $V = W \oplus W^\perp$.
   e) Basis for $W^\perp$
   f) $row(A) = N(A)^\perp$, and $col(A) = N(A^T)^\perp$
   g) $proj_W u$ given an orthogonal basis for W
   h) Gram-Schmidt Orthogonalization
4. Eigenvalues and Eigenspaces of $A \in \mathbb{R}^{n\times n}$.
   a) Basis for eigenspace $E_\lambda(A)$, algebraic and geometric multiplicity.
   b) Basic Theorems which guarantee diagonizability of A
   c) Diagonalizing $A = P\Lambda P^{-1}$
   d) Orthogonally Diagonalizing $A = Q\Lambda Q^T$ when $A = A^T$.
   e) Solving $x_n = A^n x_0$.
5. General Linear Transformations: $T : X \to Y$
   a) Finding kernel $ker(T)$ and range $R(T)$ (and dimension)
   b) Generating matrices relative to coordinates.
   $$A(u)_B = (T(u))_{B'}$$