Math 333 (2004) Assignment 1

(Due: September 14,2004 in class)

1. (10) Let $u = (1, 2, -1)$ and $v = (2, -1, -3)$. Compute the following:

a) $\parallel 2u + v \parallel$ b) $u \cdot v + \parallel u \parallel$
c) $d(u, v)$ d) $\parallel \frac{u}{\|u\|} \parallel$

2. (5) Find $u \cdot v$ if $\parallel u + v \parallel = 1$ and $\parallel u - v \parallel = 5$ (Hint: first square the norms and evaluate using dot products).

3. (5) Let $u, v, w \in \mathbb{R}^n$. Solve $3u - v = w$ for u stating which item in Theorem 4.1.1 (text) you used at each step in your calculation.

4. (15) Let $u, v, w \in \mathbb{R}^n$ and then indicate whether the following statements are true or false. When true, justify your answer with a simple logical argument. When false, give a counterexample.

a) If $\parallel u + v \parallel^2 = \parallel u \parallel^2 + \parallel v \parallel^2$ then u and v are orthogonal.

b) If u is orthogonal to v and w then u is orthogonal to $v + w$.

c) If u is orthogonal to $v + w$ then u is orthogonal to v and w.

5. (10) For each of the following, find the standard matrix $[T]$ for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

a)

$$\begin{aligned} w_1 &= 2x_1 + 3x_2 \\ w_2 &= -x_1 + x_2 - x_3 \\ w_3 &= -x_1 \end{aligned}$$

b)

$$T(x_1, x_2, x_3) = (x_3, x_2 - 2x_3, x_3 - x_1)$$

In each case compute $T(x)$ if $x = (2, 2, 1)$.

6. (30) In each of the following define $[T]$ and then compute $T(y)$ for the indicated vector y.

a) $T(x)$ is the counterclockwise rotation of $x \in \mathbb{R}^2$ by $30°$. $y = (1, 1)$.

b) $T(x)$ is the reflection about the x_1x_2 plane <u>followed</u> by a projection onto $x_2 = 0$ plane. $y = (7, 6, 5)$.

c) $T(x)$ is the counterclockwise rotation about the positive x_1 axis by $45°$ (See Table 7) followed by a reflection about the x_2x_3 plane. $y = (1, 1, 1)$.