Math 333 (2004) Assignment 3

(Due: October 7, 2004 in class)
Maximum 90points

1. (20) State which of the following sets V are vector spaces. If it is not a vector space, state <u>all</u> the axioms in pg 204 of the textbook that are not satisfied.

a) Standard addition and scalar multiplication operations on $\mathbb{R}^2$ with:

$$V = \left\{x \in \mathbb{R}^2 : x_1 \geq 0\right\}$$

b) Matrix addition and scalar multiplication and

$$V = \left\{u \in M_{22} : u = \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} \, for\ some\ a, b \in \mathbb{R}\right\}$$

c) Matrix addition and scalar multiplication and

$$V = \left\{u \in M_{22} : u = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \, for\ some\ a, b \in \mathbb{R}\right\}$$

d) The set $V = \mathbb{R}^2$ where if $x, y \in V$ and $\alpha \in \mathbb{R}$ addition and scalar multiplication are defined by:

$$\begin{aligned} x \oplus y &= (x_1 y_1, x_2 y_2) \\ \alpha \odot x &= (\alpha x_1, \alpha x_2) \end{aligned}$$

2. (35) Use Theorem 5.2.1 of the textbook to determine whether the sets W defined below are subspaces of the indicated vector spaces V. In all, the usual addition and scalar mutliplication definitions apply. When W is not a subspace indicate why.

a) $W = \left\{u \in M_{22} : u = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \, where\ a + b + c + d = 0\right\} \quad , V = M_{22}$

b) $W = \left\{u \in M_{22} : u = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \, where\ det(u) = 0\right\} \quad , V = M_{22}$

c) $W = \left\{u \in \mathbb{R}^3 : u_3 = u_1 + u_2\right\} \quad , V = \mathbb{R}^3$

d) $W = \{f \in C(\mathbb{R}) : f(x) \leq 0\} \quad , V = C(\mathbb{R})$

e) $W = \left\{f \in C(\mathbb{R}) : \frac{df}{dx} + f = 0\right\} \quad , V = C(\mathbb{R})$

f) $W = \{f \in C(\mathbb{R}) : f(0) = 0\} \quad , V = C^1(\mathbb{R})$ (be careful here)

g) $W = \left\{A \in M_{nn} : A^T = -A\right\} \quad , V = M_{nn}$

3. (20) For each of the following express w as a linear combination of the vectors v_k indicated. The vector space V which w, v_k are elements of are also indicated.

a) $V = \mathbb{R}^3$, $w = (7, 8, 9)$, $v_1 = (2, 1, 4)$, $v_2 = (1, -1, 3)$, $v_3 = (3, 2, 5)$.

b) $V = P_2$, $w = 5x^2 + 13x + 3$, $v_1 = x^2 + 2x + 3$, $v_2 = -x^2 - 3x + 1$.

c) $V = C(\mathbb{R})$, $w = cos(2x)$, $v_1 = cos^2 x$, $v_2 = sin^2 x$. (Think trig)

d) $V = M_{22}$, $w = \begin{bmatrix} -1 & -5 \\ 4 & 1 \end{bmatrix}$, $v_1 = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

4. (15) Indicate whether the following statements are true or false. When false, give a counterexample.

a) Let $A\mathbf{x} = \mathbf{b}$ be a consistent system where $A \in \mathbb{R}^{n\times n}$ and $\mathbf{b} \neq \mathbf{0}$. Then,

$$V = \{x \in \mathbb{R}^n : Ax = b\}$$

is a subspace of $\mathbb{R}^n$.

b) If $span\{v_1, v_2, \ldots v_M\} = span\{u_1, u_2, \ldots u_N\}$ then $M = N$.

c) Let $x, y, z, w \in \mathbb{R}^2$. Suppose that $w \in span\{x, y\}$ and $w \in span\{y, z\}$ then $\mathbb{R}^2 = span\{x, y, z\}$.