

## Math 333 (2004) Assignment 5

(Due: October 28, 2004 in class)

Maximum 50 points

1. (10) Let  $b = (2, 0, 0)^T$  and

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

Determine if  $b \in \text{col}(A)$  and if it is, express it as a linear combination of the columns of  $A$ .

2. (5) Find the vector form of the general solution of the linear system

$$\begin{aligned} x_1 - 3x_2 &= 1 \\ 2x_1 - 6x_2 &= 2 \end{aligned}$$

3. (15) For the matrix  $A$  defined below, find a basis for  $\text{row}(A)$ ,  $N(A)$  and  $\text{col}(A)$ .

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

4. (10) For each of the following sets of vectors  $S \subset V$ , find a basis for  $W = \text{span}(S)$  which contains only elements of  $S$

a)  $V = \mathbb{R}^4$  ,  $S = \{(1, 1, 0, 0)^T, (0, 0, 1, 1)^T, (-2, 0, 2, 2)^T, (0, -3, 0, 3)^T\}$

b)  $V = P_3$  ,  $S = \{x^3 + 1, 4 - x, 3x^3 + x, x^3 + 2\}$

5. (10) Determine a basis for  $W = \text{span}(S) \subset M_{22}$  where  $S$  is defined below. Do so by determining a basis for  $\text{row}(A)$  where  $A$  is the matrix having the coordinates of  $v_k$  relative to the standard basis  $E$  of  $M_{22}$  (not as column vectors).

$$S = \{v_1, v_2, v_3\} = \left\{ \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 8 & 1 \\ 7 & 3 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \right\}$$