Math 333 (2004) Assignment 7

(Due: November 18, 2004 in class)

Maximum 25 points

1. (15) For each of the indicated inner product spaces V, subspaces W and vector u, compute the projection $w = proj_W u$ and $w^\perp$ in the orthogonal decomposition

$$u = w + w^\perp$$

a)

$$\begin{aligned} V &= \mathbb{R}^2 \quad , \quad < u, v > \equiv (Au)^T (Av) \quad , \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} \\ u &= (1,1) \quad , \quad W = span\{(3,1)\} \end{aligned}$$

b)

$$\begin{aligned} V &= P_3 \quad , \quad < u, v > \equiv \int_0^1 u(x)v(x)dx \quad , \\ u &= x^3 - x \quad , \quad W = span\{x^3, x^2 + x\} \end{aligned}$$

c)

$$\begin{aligned} V &= M_{22} \quad , \quad < u, v > \equiv Tr(u^T v) \quad , \\ u &= \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \quad , \quad W = span\left\{ \begin{bmatrix} -1 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \right\} \end{aligned}$$

2. (5) Let $V = P_2$, $E = \{u_1, u_2, u_3\} = \{1, x, x^2\}$ be the standard basis vectors and

$$< u, v > \equiv \int_{-1}^1 u(x)v(x)dx$$

Use the Gram-Schmidt orthogonoalization process to determine an orthogonal basis from E (do not normalize the resulting vectors).

3. (5) Determine an <u>orthonormal</u> basis $S = \{v_1, v_2, v_3\}$ for $V = \mathbb{R}^3$ which contains only vectors in $row(A)$ and $N(A)$ if

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$