MATH 333: Midterm
October 19, 2004. Name:______________________________

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1. [30] For each of the linear transformations T defined below, determine the standard matrix $[T]$ associated with it.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(\mathbf{x}) \equiv (2x_3 - x_1, x_1 + 5x_2)$ where $\mathbf{x} = (x_1, x_2, x_3)$.

$[T] =$

(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where $T(\mathbf{x})$ is the projection of $\mathbf{x}$ onto $\mathbf{u} = (3, -1)$.

$[T] =$

(c) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where if $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are the standard basis vectors and

$$T(\mathbf{e}_1 - \mathbf{e}_3) = (2, 1, 2)\ ,\ T(\mathbf{e}_1 + \mathbf{e}_3) = (0, 1, 2)\ ,\ T(\mathbf{e}_2) = (3, 1, 2)\ .$$

$[T] =$

2. [20] Indicate which of the following sets W are subspaces of the indicated vector spaces V. If W is not a subspace, state all closure properties which are not satisfied and illustrate their failure with a specific example.

a) $V = \mathbb{R}^3$, $W = \{x \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 1\}$ where $x = (x_1, x_2, x_3)$.

b) $V = P_2$, $W = \{u \in P_2 : u(2) = 0\}$ where $u(x) = ax^2 + bx + c$ for scalars a, b, c.

c) $V = M_{22}$, $W = \{A \in M_{22} : Tr(A) = a_{11} + a_{22} = 0\}$ where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

d) $V = P_2$, $W = \{u \in P_2 : u(x) = ax^2 + bx + c\ ,\ a > 0\}$.

3. [10] Compute a basis S for the solution space of the following system of equations:

$$x_1 + x_2 + x_3 + x_4 = 0 \tag{1}$$

$$2x_1 - x_2 + 2x_3 - x_4 = 0\ . \tag{2}$$

$S =$

4. [10] Let $w = (1, 1)$, $v_1 = (2, 1)$ and $v_2 = (-1, 3)$ be vectors in $\mathbb{R}^2$. Write w as a linear combination of v_1 and v_2, that is, find scalars a, b such that

$$w = av_1 + bv_2$$

5. [10] Let $V = P_2$ and $w = 12x \in V$. Compute the coordinate $(w)_S = (c_1, c_2, c_3)$ of w relative to $S = \{v_1, v_2, v_3\}$ where

$$v_1 = 2x^2 + x - 1 \quad , \quad v_2 = x^2 + 2x + 1 \quad , \quad v_3 = -x^2 + 3x \ .$$

6. [20] Indicate whether the following statements are True (T) or False (F). When the statement is False, give a simple counterexample.

a) Let $u, v, w \in \mathbb{R}^3$. If u is orthogonal to v and w then u is orthogonal to $2v - w$.

b) Two independent vectors $x, y \in \mathbb{R}^3$ span every two dimensional subspace W of $\mathbb{R}^3$.

c) Let V be a vector space with $dim(V) = 3$ and $S = \{u_1, u_2, u_3, u_4\} \subset V$ where u_1, u_2, u_3 are independent. Then $V = span(S)$.

d) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation then $dim(N(A)) = 0$ where $A = [T]$.