Math 451 (2018) – Homework 9
(THROUGHOUT USE INTEGRAL TABLES AND ODE SOLVERS AS NEEDED)

Due: Friday, March. 23, 2018.   NAME: _____

1. [10 pts] Define the integral operator on $L^2[0,1]$:

$$Ku \equiv \int_0^1 (12xy + 6y^2)u(y)\ dy$$

a) Solve the integral equation

$$\int_0^1 (12xy + 6y^2)u(y)\ dy - \lambda u(x) = f(x)$$

where $\lambda = 3$ and $f(x) = 20$.

b) Find the largest eigenvalue λ_+ of K and its associated eigenfunction $\phi_+(x)$ (nonunique). The former is an eigenvalue of the matrix A in the notes. The latter has the form $\phi_+(x) = ax + b$ for certain contants a, b.

2. [10 pts] Find the Green's functions associated with the following boundary value problems:

a)

$$
\begin{aligned}
u'' + u &= f(x) \\
u(0) &= 0 \quad , \quad u(\pi) + u'(\pi) = 0
\end{aligned}
$$

b)

$$
\begin{aligned}
-u'' + \frac{2}{x^2}u &= f(x) \\
u(1) &= 0 \quad , \quad u'(2) = 0
\end{aligned}
$$