Math 450 (2017) – Homework 1

Due: Wed. September 13, 2017. NAME: _____

1. [20pts] Find the solution of the following initial value problems:

$$y' - \frac{1}{2x}\, y = \sqrt{x} \quad , \quad y(1) = 4 \tag{1}$$

$$y' + \frac{1}{x}\, y = \frac{5x}{y^2} \quad , \quad y(1) = 2 \tag{2}$$

$$y'' - 2y' + 5y = 0 \quad , \quad y(0) = 0 \quad , \quad y'(0) = 1 \tag{3}$$

$$y'' + 2y' + y = 1 \quad , \quad y(0) = 1 \quad , \quad y'(0) = 2 \tag{4}$$

2. [5pts] Use the method of Variation of Parameters to find a particular solution of

$$y'' - 2y' + y = x^3 e^x$$

3. [5pts] Let $f(x, y) = y - \log(x)$. Find that unique curve through $(x, y) = (1, 0)$ that is orthogonal to the level curves of f. Sketch several $f = c$ level curves and the resulting orthogonal curve (just for $x > 0$).

4. [10pts] Find a Fundamental Matrix $X(t) \in \mathbb{R}^{2\times 2}$ of the system

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

for the following two matrices:

$$A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \qquad , \qquad A = \begin{bmatrix} 2 & -5 \\ 1 & 0 \end{bmatrix}$$