Math 451 (2018) – Homework 10

(THROUGHOUT USE INTEGRAL TABLES AND ODE SOLVERS AS NEEDED)

Due: Friday April 13, 2018. NAME: ____________________

1. [9 pts] Use the properties of distributions to show

a) $x\delta'(x) = -\delta(x)$

b) $\alpha(x)\delta'(x) = -\alpha'(0)\delta(x) + \alpha(0)\delta'(x)$ where $\alpha \in C_0^\infty(\mathbb{R})$.

c) $x\delta'''(x) = -3\delta''(x)$

2. [6 pts] Let $I = [0,1]$ and $\psi(x,\zeta)$ be any function such that

$$\frac{d^2\psi}{dx^2} = 0 \quad , \quad \forall x \in I$$

Define

$$g(x,\zeta) = \frac{1}{2}|x-\zeta| + \psi(x,\zeta)$$

a) Show that $g(x,\zeta)$ is the distribution solution of

$$\frac{d^2 g}{dx^2} = \delta(x-\zeta)$$

Here you need to show $< g'', \phi > \equiv < g, \phi'' > = \phi(\zeta)$ for all test functions ϕ.

b) The general solution for ψ is $\psi = A(\zeta)x + B(\zeta)$ where A, B are some functions of ζ. Find the function ψ so that

$$g(0,\zeta) = 0 \qquad , \qquad g_x(1,\zeta) = 0$$

The resulting function $g(x,\zeta)$ is the Green's function for $Lu = u''$ on $I = [0,1]$ satisfying the above boundary conditions.