Math 451 (2018) – Homework 6 (max=30)

Due: Friday, January 26, 2018. NAME: ____________________

1. [14 pts] Find <u>all</u> the natural boundary conditions associated with extremizing $J(y)$ over $\mathcal{A}$ for the following functionals and admissible sets.

(a)

$$\begin{aligned} J(y) &= y(1)^3 + \int_0^1 \left(y^2 - (x+1)^3 y'^2\right)\, dx \\ \mathcal{A} &= \{y :\ y \in C^2[0,1], y(0) = 1\} \end{aligned}$$

(b)

$$\begin{aligned} J(y) &= \int_0^1 (yy'' + xy')\, dx \\ \mathcal{A} &= \{y :\ y \in C^4[0,1], y(0) = 1\} \end{aligned}$$

Do not find the extrema. Just derive all the natural boundary conditions. It is easiest if one derives the conditions using a general Lagrangian. For example, for part (b) start with

$$\delta J(y,h) = \left(hL_{y'} + h'L_{y''} - h\frac{d}{dx}L_{y''}\right)\Bigg|_{x=0}^{x=1} + \int_0^1 \left(L_y - \frac{d}{dx}L_{y'} + \frac{d^2}{dx^2}L_{y''}\right) dx$$

for an arbitrary Lagrangian L.

2. [8 pts] Find the extrema of

$$J(y) \equiv \int_0^1 \left(\frac{1}{2}y'^2 + y'y + y' + y\right)\, dx$$

over

$$\mathcal{A} = \{y :\ y \in C^2[0,1],\ y(0) = \frac{1}{2}\}$$

3. [8 pts] Find the extrema of

$$J(y) \equiv \int_0^1 \left(yy' + (y'')^2\right)\, dx$$

over

$$\mathcal{A} = \{y :\ y \in C^4[0,1],\ y(0) = 0, y'(0) = 1, y(1) = 2, y'(1) = 4\}$$