§7.2 # 1, 2, 3, 5, 9, 15, 17, 23, 24, 29, 33, 35, 41, 43, 44, 45, 47, 51, 53, 57, 59

1. $\int \cos^3 x \; dx = \int \cos^2 x \cos x \, dx = \int (1-\sin^2 x) \cos x \, dx = \int (1-u^2)\, du = u - \frac{u^3}{3} + C = \sin x - \frac{1}{3}\sin^3 x + C$ ✓

$u = \sin x$

$du = \cos x \, dx$

2. $\int \sin^5 x \; dx = \int (\sin^2 x)^2 \sin x \, dx = \int (1-\cos^2 x)^2 \sin x \, dx = -\int (1-u^2)^2 du = -\int (1-2u^2+u^4) du = -(u - \frac{2}{3}u^3 + \frac{1}{5}u^5) + C$

$u = \cos x$

$du = -\sin x \, dx$

$= \frac{2}{3}\cos^3 x - \cos x - \frac{1}{5}\cos^5 x + C$

3. $\int \sin^3\theta \cos^2\theta \, d\theta = \int (1-\cos^2\theta)\cos^2\theta \sin\theta \, d\theta = -\int (1-u^2)u^2 du = \int (u^4 - u^2) du = \frac{1}{5}u^6 - \frac{1}{3}u^3 + C$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$= \frac{1}{5}\cos^5\theta - \frac{1}{3}\cos^3\theta + C$ ✓

5. $\int \sin^3 t \cos^3 t \, dt = \int \sin^3 t (1-\sin^2 t)\cos t \, dt = \int u^3(1-u^2) du = \frac{1}{4}u^4 - \frac{1}{6}u^6 + C = \frac{1}{4}\sin^4 t - \frac{1}{6}\sin^6 t + C$ ✓

$u = \sin t$

$du = \cos t \, dt$

9. $\int \cos^4 y \; dy = \int (\frac{1}{2}(1+\cos 2y))^2 dy = \frac{1}{4}\int (1 + 2\cos 2y + \underbrace{\frac{1}{2}(1+\cos 4y)}_{= \cos^2 2y}) dy = \frac{1}{4}[y + \sin 2y + y/2 + 1/8 \sin 4y] + C$

$= \frac{3y}{8} + \frac{1}{4}\sin 2y + \frac{1}{32}\sin 4y + C$

15. $\int \tan^3 x \sec x \, dx = \int \tan^2 x \tan x \sec x \, dx = \int (\sec^2 x - 1)\tan x \sec x \, dx = \int (u^2-1) du = \frac{1}{3}u^3 - u + C$

$u = \sec x$

$du = \sec x \tan x \, dx$

$= \frac{1}{3}\sec^3 x - \sec x + C$ ✓

17. $\int \tan^2 x \sec^4 x \, dx = \int \tan^2 x (1+\tan^2 x)\sec^2 x \, dx = \int u^2(1+u^2) du = \frac{1}{3}u^3 + \frac{1}{5}u^5 + C$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$ ✓

23. $\int \cos^5 x \sin x \; dx = -\int u^5 du = -\frac{1}{6}u^6 + C = C - \frac{1}{6}\cos^6 x$ ✓

$u = \cos x$

$du = -\sin x \, dx$

24. $\int \cos^3(2-x)\sin(2-x)\, dx = \int u^3 du = \frac{1}{4}u^4 + C = \frac{1}{4}\cos^4(2-x) + C$ ✓

$u = \cos(2-x)$

$du = \sin(2-x) dx$

§7.2 # 29, 33, 35, 41, 43, 44, 45, 47, 51, 53, 57, 59

29. $\int \sin^4 3x \, dx = \int \left[\frac{1}{2}(1-\cos 6x)\right]^2 dx = \frac{1}{4}\int (1-2\cos 6x + \cos^2 6x)\, dx = \frac{1}{4}\int \left(1 - 2\cos 6x + \frac{1}{2}(1+\cos 12x)\right) dx$

$= \frac{1}{4}\left[x - \frac{1}{3}\sin 6x + \frac{x}{2} + \frac{1}{24}\sin 12x\right] + C = \frac{3x}{8} - \frac{1}{12}\sin 6x + \frac{1}{96}\sin 12x + C$

33. $\int \tan x \sec^2 x \, dx = \int u \, du = \frac{1}{2}u^2 + C = \frac{1}{2}\tan^2 x + C$

$u = \tan x$

$du = \sec^2 x \, dx$

35. $\int \tan^5 x \sec^4 x \, dx = \int \tan^5 x (1+\tan^2 x)\sec^2 x \, dx = \int (u^5 + u^7)\, du = \frac{1}{6}u^6 + \frac{1}{8}u^8 + C$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \frac{1}{6}\tan^6 x + \frac{1}{8}\tan^8 x + C$

41. $\int \sin 2x \cos 2x \, dx = \frac{1}{2}\int u \, du = \frac{1}{4}u^2 + C = \frac{1}{4}\sin^2 2x + C$

$u = \sin 2x$

$du = 2\cos 2x \, dx$

43. $\int t \cos^3 t^2 \, dt = \frac{1}{2}\int \cos^3 u \, du = \frac{1}{2}\int (1-\sin^2 u)\cos u \, du = \frac{1}{2}\int (1-x^2)\, dx = \frac{1}{2}\left(x - \frac{1}{3}x^3\right) + C$

$u = t^2$

$du = 2t \, dt$

$x = \sin u$

$dx = \cos u \, du$

$= \frac{1}{2}\sin(t^2) - \frac{1}{6}\sin^3(t^2) + C$

44. $\int \frac{\tan^3(\ln t)}{t}\, dt = \int \tan^3 x \, dx = \int (\sec^2 x - 1)\tan x \, dx = \int \sec^2 x \tan x \, dx - \int \tan x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = \ln t$

$dx = \frac{1}{t}\, dt$

$= \int u \, du - \int \tan x \, dx = \frac{1}{2}\tan^2 x - \ln|\sec x| + C$

$= \frac{1}{2}\tan^2(\ln t) - \ln|\sec(\ln t)| + C$

45. $\int \cos^2(\sin t)\cos t \, dt = \int \cos^2 x \, dx = \frac{1}{2}\int (1+\cos 2x)\, dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$

$x = \sin t$

$dx = \cos t \, dt$

$= \frac{1}{2}\sin t + \frac{1}{4}\sin(2\sin t) + C$

§7.2 # 47, 51, 53, 57, 59

47. $$\int_0^{2\pi} \sin^2 x \, dx = \frac{1}{2}\int_0^{2\pi} (1 - \cos 2x)\, dx = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\Big|_0^{2\pi} = \pi$$

51. $$\int_0^{\pi/4} \frac{dx}{\cos x} = \int_0^{\pi/4} \sec x \, dx = \ln|\sec x + \tan x|\Big|_0^{\pi/4} = \ln|\sqrt{2}+1| - \cancelto{0}{\ln|1+0|} = \ln(1+\sqrt{2})$$

53. $$\int_0^{\pi/3} \tan x \, dx = \ln|\sec x|\Big|_0^{\pi/3} = \ln(2) - \cancelto{0}{\ln(1)} = \ln 2$$

$\sin(-x) = -\sin x$

57. $$\int_0^{\pi} \sin 3x \cos 4x \, dx = \int_0^{\pi} \frac{1}{2}[\sin 7x + \sin(-x)]\, dx = \frac{1}{2}\int_0^{\pi} (\sin 7x - \sin x)\, dx$$

$$= \frac{1}{2}\left[-\frac{1}{7}\cos 7x + \cos x\right]\Big|_0^{\pi} = \frac{1}{2}\left[\left(\frac{1}{7} - 1\right) - \left(-\frac{1}{7} + 1\right)\right] = -6/7$$

59. $$\int_0^{\pi/6} \sin 2x \cos 4x \, dx = \int_0^{\pi/6} \frac{1}{2}[\sin 6x + \sin(-2x)]\, dx = \frac{1}{2}\int_0^{\pi/6} (\sin 6x - \sin 2x)\, dx$$

$$= \frac{1}{2}\left[-\frac{1}{6}\cos 6x + \frac{1}{2}\cos 2x\right]\Big|_0^{\pi/6} = \frac{1}{2}\left[\left(\frac{1}{6} + \frac{1}{4}\right) - \left(-\frac{1}{6} + \frac{1}{2}\right)\right] = \frac{1}{2}\left[\frac{2+3+2-6}{12}\right]$$

$$= \frac{1}{24}$$ ⟸ Note: solution in text is wrong.

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

so $\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$