

5. [15pts] Conversion Problems: answer all three questions below.

a) Find an equation $y = f(x)$ for the parametric curve $c(t) = \left(\frac{1}{t-1}, \frac{1}{t}\right)$.

$$y = \frac{1}{t} \Rightarrow t = \frac{1}{y} \quad \underline{\text{eliminate } t}$$

$$x = \frac{1}{\frac{1}{y} - 1}$$

$$\frac{1}{x} = \frac{1}{y} - 1 \quad \underline{\text{solve for } x}$$

$$y = f(x) = \underline{\left(\frac{1}{x} + 1\right)^{-1}}$$

b) Convert the polar equation $r = \sec \theta$ to an equation in rectangular (Cartesian) coordinates.

$$r = \frac{1}{\cos \theta}$$

$$r \cos \theta = 1$$

$$\boxed{x = 1}$$

c) Find a polar equation of the circle $(x-1)^2 + y^2 = 1$, i.e. find an $f(\theta)$ such that $r = f(\theta)$.

$$x^2 - 2x + 1 + y^2 = 1$$

$$r^2 - 2r \cos \theta = 0$$

$$r(r - 2 \cos \theta) = 0$$

$$r \neq 0$$

$$r = 2 \cos \theta$$

$$r = f(\theta) = \underline{2 \cos \theta}$$