

Practice Integration Problems MATH 172:

The integrals practice problems on the following pages can all be evaluated using combinations of

- 1) The Method of Substitution
 - 2) Integration by Parts
 - 3) Trigonometric identities
 - 4) Inverse Trigonometric Substitutions
 - 5) Partial fraction expansions
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Some commonly used trigonometric identities are:

$$\begin{aligned}\sin^2(x) + \cos^2(x) &= 1 \\ \tan^2(x) + 1 &= \sec^2(x) \\ \cos^2(x) &= \frac{1}{2}(1 + \cos(2x)) \\ \sin^2(x) &= \frac{1}{2}(1 - \cos(2x)) \\ \sin(2x) &= 2 \sin(x) \cos(x) \\ \sin(x) \cos(y) &= \frac{1}{2}(\sin(x+y) + \sin(x-y)) \\ \cos(x) \cos(y) &= \frac{1}{2}(\cos(x+y) + \cos(x-y)) \\ \sin(x) \sin(y) &= \frac{1}{2}(\cos(x-y) - \cos(x+y))\end{aligned}$$

Some commonly integrals worth noting include:

$$\begin{aligned}\int \frac{1}{u^2 + 1} du &= \arctan(u) + c \\ \int \frac{1}{\sqrt{1-u^2}} du &= \arcsin(u) + c \\ \int \tan(u) du &= \ln|\sec(u)| + c \\ \int \sec(u) du &= \ln|\sec(u) + \tan(u)| + c\end{aligned}$$

(I) Quickies:

$$\begin{aligned}a) \int 5e^{3x} dx &= \frac{5}{3}e^{3x} + c & b) \int 2\cos(\pi x) dx &= \frac{2}{\pi}\sin(\pi x) + c \\ c) \int \sec(2x) \tan(2x) dx &= \frac{1}{2}\sec(2x) + c & d) \int 7\sec^2(5x) dx &= \frac{7}{5}\tan(5x) + c \\ e) \int \frac{dx}{x^2+4} &= \frac{1}{2}\arctan\left(\frac{x}{2}\right) + c & f) \int \frac{x}{x^2+1} dx &= \frac{1}{2}\ln|x^2+1| + c \\ g) \int \frac{2}{3x+1} dx &= \frac{2}{3}\ln|3x+1| + c\end{aligned}$$

(II) Intermediate Difficulty Problems:

1) $\int_1^2 \frac{\ln(x)}{x} dx$

3) $\int \frac{2x+1}{x(1-x)} dx$

5) $\int \frac{e^{\sqrt{z}}}{\sqrt{z}} dz$

7) $\int \frac{dx}{\sqrt{9-x^2}}$

9) $\int \frac{3x+2}{x^2(x+2)} dx$

11) $\int \frac{3x^3+x^2+4}{3x+1} dx$

13) $\int \frac{1}{\sqrt{x(\sqrt{x}+1)}} dx$

15) $\int e^{3x} \cos(4x) dx$

17) $\int_0^1 \arcsin(x) dx$

19) $\int \sqrt{x^2-4} dx$ *needtable*

21) $\int \frac{4x+7}{(x+1)(2x+3)} dx$

23) $\int \frac{\sin(\ln(x))}{x} dx$

25) $\int \sec^4(x) dx$

27) $\int \cos^2(4x) dx$

29) $\int_{\pi/4}^{\pi/3} \frac{\sec^2(x)}{\tan(x)} dx$

31) $\int \frac{1}{x^2+4x+5} dx$

33) $\int \frac{4x^2-2x}{(x-1)(x^2+1)} dx$

35) $\int x^2 \ln(x) dx$

37) $\int \tan(x) \sec^3(x) dx$

39) $\int \frac{2x+1}{x^2-1} dx$

41) $\int \frac{1}{\sqrt{x^2+2x+2}} dx$

2) $\int_1^2 \frac{\ln(x)}{x^2} dx$

4) $\int x e^{x/2} dx$

6) $\int \tan^3(x) \sec^2(x) dx$

8) $\int \frac{dx}{\sqrt{x^2-9}}$

10) $\int \frac{x^3}{1+x^4} dx$

12) $\int_0^{\pi/2} \sin^2(x) dx$

14) $\int (2x+1) \cos(x) dx$

16) $\int \frac{1}{x\sqrt{1+x^2}} dx$

18) $\int_0^{\pi/6} \frac{\cos(x)}{1+\sin(x)} dx$

20) $\int \sqrt{4-x^2} dx$

22) $\int \frac{x}{\sqrt{1+x^2}} dx$

24) $\int x^2 e^x dx$

26) $\int \frac{e^x}{e^{2x}+1} dx$

28) $\int \cos^2(x) \sin^3(x) dx$

30) $\int \arctan(2x) dx$

32) $\int \sin(2x) \cos(4x) dx$

34) $\int \frac{1}{(x^2+4)^{3/2}} dx$

36) $\int x^2 e^{x^3} dx$

38) $\int \frac{x}{\sqrt{1+x^2}} dx$

40) $\int \sin(x) \cos^3(x) dx$

42) $\int \frac{3 \cos(x)}{\sqrt{1+3\sin(x)}} dx$

Answers:

- 1) $u = \ln(x)$; $\frac{1}{2}(\ln(2))^2$
- 3) *partial fraction* ; $\ln|x| - 3\ln|x - 1| + c$
- 5) $u = \sqrt{x}$; $2e^{\sqrt{x}} + c$
- 7) $u = x/3$; $\arcsin(x/3) + c$
- 9) *partial fractions*; $\ln|x| - \ln|x + 2| - x^{-1} + c$
- 11) *Long division*; $1/3x^3 + 4/3 \ln|3x + 1| + c$
- 13) $u = \sqrt{x} + 1$; $2\ln(1 + \sqrt{x}) + c$
- 15) *IBParts twice*; $3/25e^{3x}\cos(4x) + 4/25e^{3x}\sin(4x) + c$
- 17) $u = \arcsin(x)$, $v = x$; $\pi/2 - 1$
- 19) $x = 2\sec(\theta)$; $1/2x\sqrt{x^2 - 4} - 2\ln|x + \sqrt{x^2 - 4}| + c$
- 21) *partial fraction*; $3\ln|x + 1| - \ln|2x + 3| + c$
- 23) $u = \ln(x)$; $-\cos(\ln|x|) + c$
- 25) *trig.ident.* then $u = \tan(x)$; $\tan(x) - 1/3\tan^3(x) + c$
- 27) *trig. ident.*; $1/2x + 1/6 * \sin(8x) + c$
- 29) $u = \tan(x)$; $1/2\ln(3) = \ln(\sqrt{3})$
- 31) $u = x + 2$, *complete square*; $\arctan(x + 2) + c$
- 33) *Partial. Frac.*; $\ln|x - 1| + \arctan(x) + 3/2\ln(x^2 + 1) + c$
- 35) *IBP* $u = \ln(x)$, $v = 1/3x^3$; $1/3x^3\ln|x| - 1/9x^3 + c$
- 37) *trig.* $u = \sec(x)$; $1/3 \sec^3(x) + c$
- 39) *partial frac.*; $3/2\ln|x - 1| + 1/2\ln|x + 1| + c$
- 41) $x + 1 = \tan(\theta)$; $\ln|x + 1 + \sqrt{x^2 + 2x + 2}| + c$
- 2) $u = \ln(x)$, $v = -x^{-1}$; $1/2(1 - \ln(2))$
- 4) $u = x$, $v = 2e^{x/2}$; $2(x - 2)e^{x/2} + c$
- 6) $u = \tan(x)$; $1/4 \tan^4(x) + c$
- 8) $x = 3 \sec(\theta)$; $\ln|x + \sqrt{x^2 - 9}| + c$
- 10) $u = 1 + x^4$; $1/4 \ln(x^4 + 1) + c$
- 12) *trig ident.*; $\pi/4$
- 14) $u = 2x + 1$, $v = \sin(x)$; $2\cos(x) + (2x + 1) \sin(x) + c$
- 16) $x = \tan(\theta)$; $-\ln|(1 + \sqrt{1 + x^2})/x| + c$
- 18) $u = 1 + \sin(x)$; $\ln(3) - \ln(2)$
- 20) $x = 2\sin(\theta)$; $1/2x\sqrt{4 - x^2} + 2\arcsin(x/2) + c$
- 22) $u = 1 + x^2$; $\sqrt{1 + x^2}$
- 24) *IBP twice*, $u = x^2$, $v = e^x$; $(2 - 2x + x^2)e^x$
- 26) $u = e^x$; $\arctan(e^x) + c$
- 28) *trig.ident.* $u = \cos(x)$; $1/5\cos^5(x) - 1/3\cos^3(x) + c$
- 30) *IBP* $u = \arctan(2x)$, $v = x$; $x \arctan(2x) - 1/4\ln(1 + 4x^2) + c$
- 32) *trig. ident.*; $1/4\cos(2x) - 1/12\cos(6x) + c$
- 34) $x = 2\tan(\theta)$; $x/(4\sqrt{x^2 + 4}) + c$
- 36) $u = x^3$; $1/3e^{x^3} + c$
- 38) $u = 1 + x^2$; $\sqrt{1 + x^2} + c$
- 40) $u = \cos(x)$; $-1/4\cos^4(x) + c$
- 42) $u = 1 + 3\sin(x)$; $2\sqrt{1 + 3\sin(x)} + c$