Math 172 - Sequence and Series Review Problems

1. Determine if the following series converge or diverge. Determine the sum for convergent series.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{2^{2k}}$

   (b) $\displaystyle\sum_{k=0}^{\infty} 2^k 3^{2-k}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{3^{2k+1}}{2^{3k+1}}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \cos k$

   (e) $\displaystyle\sum_{k=2}^{\infty} \frac{2}{k^2-1}$

   (f) $\displaystyle\sum_{k=2}^{\infty} \frac{6}{4k^2-9}$

2. Use the integral test to determine whether the following converge. Make sure you state/verify the hypotheses.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{e^{n^3}}$

   (c) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n \ln n}$

3. Determine if the following series converge or diverge.

   (a) $\displaystyle\sum \frac{n+2}{n^4}$

   (b) $\displaystyle\sum \frac{\cos(\pi n)\ln n}{n}$

   (c) $\displaystyle\sum \frac{\sqrt[4]{n^3-1}}{n^2+n}$

   (d) $\displaystyle\sum \frac{2^n}{3^n+4^n}$

   (e) $\displaystyle\sum \frac{2n+1}{3n^2+4}$

   (f) $\displaystyle\sum \left(1+\frac{(-1)^n}{n}\right)$

4. Determine if the following series absolutely converge, conditionally converge, or diverge.

   (a) $\displaystyle\sum \frac{\sin n}{n^2+1}$

   (b) $\displaystyle\sum \frac{(2n)!}{n^5 5^n n!}$

   (c) $\displaystyle\sum \frac{n^2 2^n}{(2n+1)!}$

   (d) $\displaystyle\sum \frac{(-1)^n}{n+\sqrt{n}}$

   (e) $\displaystyle\sum \left(\frac{2n+1}{3n-2}\right)^{2n}$

   (f) $\displaystyle\sum \frac{(-1)^n n^3}{(n^2+3)^2}$

5. Find the radius of convergence and interval of convergence for the following.

   (a) $\displaystyle\sum \frac{(x-1)^n}{n5^n}$

   (b) $\displaystyle\sum \frac{(2x)^n}{n^n}$

   (c) $\displaystyle\sum (5x)^n$

   (d) $\displaystyle\sum n(2-x)^n$

   (e) $\displaystyle\sum \frac{(x+4)^n}{n^4}$

   (f) $\displaystyle\sum \frac{x^n}{n!}$

6. True/False Questions. If false, provide a counterexample.

   (a) T/F If $\lim_{n\to\infty} |a_n| = 0$ then $\{a_n\}$ converges.

   (b) T/F If $\lim_{n\to\infty} |a_n| = 0$ then $\sum a_n$ converges.

   (c) T/F If $\lim_{n\to\infty} a_n = 1$ then $\{a_n\}$ converges.

   (d) T/F If $\lim_{n\to\infty} |a_n| = 1$ then $\{a_n\}$ converges.

   (e) T/F If $\lim_{n\to\infty} |a_n| = 1$ then $\sum a_n$ converges.

   (f) T/F If $\lim_{n\to\infty} a_n = \infty$ then $\{a_n\}$ converges.

   (g) T/F The power series $\sum a_n x^n$ can diverge for all values of x.

   (h) T/F The power series $\sum a_n x^n$ can converge for all values of x.

   (i) T/F If the power series $\sum a_n x^n$ converges for $x = -2$ then the series converges for $x = 1$.

   (j) T/F If $\sum a_n$ converges conditionally then $\sum |a_n|$ converges.